

Linear Combination of random variables.

For a random variable 'x'

mean $\rightarrow E(x) = \sum x_i p_i$

Variance $\rightarrow \text{Var}(x) = \sum x_i^2 p_i - [E(x)]^2$

$\rightarrow$ if $E(x+b) = E(x) + b$

if $\text{Var}(x+b) = \text{Var}(x)$

Ex. data $\rightarrow 100, 150, 200, 50, 100$

$$\bar{X} = \frac{600}{5} = 120$$

add a constant = 20

$100+20, 150+20, 200+20, 50+20, 100+20$

New mean = $\bar{X} + 20$

$\rightarrow$ if $E(ax) = a E(x)$

if $\text{Var}(ax) = a^2 \text{Var}(x)$

$\rightarrow$ if $E(ax+b) = a E(x) + b$

if $\text{Var}(ax+b) = a^2 \text{Var}(x)$

$\Rightarrow$ Questions

Q. $X \sim B(25, 0.6)$

Find $E(5X-1)$:

$\mu = 25 \times 0.6$

$= 15$

$E(5X-1) = [15 \times 5] - 1$

$= \underline{\underline{74}}$

find $\text{Var}(x)$

$\text{Var}(5X-1) = 5^2 \times 6$

$= \underline{\underline{150}}$

Q. $X \sim B(30, 0.2)$

$\mu = 6$

$E(2x+3) = \underline{15} = 2(E(x)) + 3 = 2(6) + 3 = \underline{15}$

$Var(2x+3) = \underline{19.2} = 2^2(E(x)) = 4(4.8) = \underline{19.2}$

Q. $E(x) = 1.3$

$Var(x) = 1.3$

Find $E(4x-1) = 4(E(x)) - 1 = 4(1.3) - 1 = \underline{5}$

Find $Var(4x-1) = 4^2(Var(x)) = 16 \times 1.3 = \underline{20.8}$

$E(ax-b) = 20$

$Var(ax-b) = 130$

$a^2 Var(x) = 130$

$a(5) - b = 20$

$1.3a^2 = 130$

$5a - b = 20$

$a^2 = 100$

$a = 10$

$5 \times 10 - b = 20$

$50 - b = 20$

$50 - 20 = b$

$b = 30$

$\Rightarrow$ Rules. If there are 2 random variables, 'X' and 'Y'

1. $E(X+Y) = E(X) + E(Y)$

$Var(X+Y) = Var(X) + Var(Y)$

2. $E(X-Y) = E(X) - E(Y)$

$Var(X-Y) = Var(X) + Var(Y)$

3. $E(ax+by) = aE(X) + bE(Y)$

$Var(ax+by) = a^2E(X) + b^2E(Y)$

⇒ Linear combination of poisson distribution

→ X and Y follows poisson distribution

then $X+Y$ follows poisson distribution

$$X \sim \text{Po}(\lambda_1), \quad Y \sim \text{Po}(\lambda_2)$$

$$X+Y \sim \text{Po}(\lambda_1 + \lambda_2)$$

Q. $X \sim \text{Po}(4.2) \quad Y \sim \text{Po}(6.3)$

Find $E(X)$ and $\text{Var}(X)$

a] $2X - Y$

$$E(2X - Y) = 2E(X) - E(Y)$$

$$2(4.2) - 6.3$$

$$= 2.1$$

$$\text{Var}(2X - Y) = 2^2(\text{Var}(X)) + \text{Var}(Y)$$

$$= 16.8 + 6.3$$

$$= \underline{\underline{23.1}}$$